Advanced Robot Control

Static linearisation

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Presentation compiled for taking notes during lecture

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Outline

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Types of description (1/4)

The mobile robot can be described using different types of description [1]:

- In generalized coordinates,
- In auxiliary velocities,
- In linearised coordinates.

Types of description (2/4)

Generalized coordinates

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + D(q) = Bu \qquad (1)$$

Types of description (3/4)

Auxiliary velocities

$$\begin{cases} M\dot{\eta} + C\eta + D = Bu, \\ \dot{q} = G\eta. \end{cases} \tag{2}$$

$$\eta, u \in R^m \tag{3}$$

Types of description (4/4)

Linearising coordinates

$$\xi = \begin{pmatrix} \xi_1 \\ \xi_2 \end{pmatrix}. \tag{4}$$

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Linearised coordinates

- ξ_1 – group of coordinates that can be linearised statically,
- ξ_2 – group of coordinates that **can not** be linearised statically.

Let

$$\begin{aligned} \xi_1 &= h(q) \in R^m, \\ \xi_2 &= k(q) \in R^{n-m}. \end{aligned}$$

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Differentiation of ξ_1

$$\xi_1 = h(q) \tag{5}$$

$$\dot{\xi}_1 = \frac{\partial h}{\partial q}\frac{dq}{dt} = \frac{\partial h}{\partial q}\dot{q} = \frac{\partial h}{\partial q}G\eta = R^{-1}\eta \tag{6}$$

The matrix R^{-1} is defined as

$$R^{-1} = \left[\frac{\partial h}{\partial q}G\right]_{m\times m} \tag{7}$$

The matrix R^{-1} is called a **decoupling matrix.**

Regularity condition

$$detR^{-1} \neq 0 \tag{8}$$

Let's assume that the above condition is fulfilled then

$$\dot{\xi}_1 = R^{-1}\eta$$

after left-hand side multiplication by R matrix we get

$$R/\ \ \dot{\xi}_1 = R^{-1}\eta$$

$$\Leftrightarrow$$

$$\eta = R\dot{\xi}_1, \tag{9}$$

$$\dot{\eta} = \dot{R}\dot{\xi}_1 + R\ddot{\xi}_1. \tag{10}$$

Reformulation of dynamic equations (1/3)

After substitution of (9)–(10) to (2) we get

$$M\dot{\eta} + C\eta + D = Bu \tag{11}$$

$$M\left(\dot{R}\dot{\xi}_1 + R\ddot{\xi}_1\right) + CR\dot{\xi}_1 + D = Bu. \tag{12}$$

After left-hand side multiplication by the transposition of R matrix we get

$$R^T / \quad MR\ddot{\xi}_1 + M\dot{R}\dot{\xi}_1 + CR\dot{\xi}_1 + D = Bu \tag{13}$$

After grouping we get

Reformulation of dynamic equations (2/3)

$$J(q)\ddot{\xi}_1 + C_\xi \dot{\xi}_1 + D_\xi = B_\xi u \tag{14}$$

where

$$\begin{aligned} J(q) &= R^T M R \\ C_\xi &= R^T \left(M\dot{R} + CR \right) \\ D_\xi &= R^T D \\ B_\xi &= R^T B \end{aligned}$$

The $J(q)$ matrix is inertia matrix expressed in linearised coordinates. Dynamics expressed in linearised coordinates has

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Reformulation of dynamic equations (3/3)

following general form

$$J\ddot{\xi}_1 + C_\xi \dot{\xi}_1 + D_\xi = B_\xi u \tag{15}$$

Property of the system

The system expressed as (15) has the same structure as a manipulator.

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Symmetry of J matrix (1/1)

It is required of the inertia matrix J to be symmetric because control algorithms require this property.

How to check if J is symmetric?

$$J \stackrel{?}{=} J^T \tag{16}$$

$$J^T = (R^T M R)^T = R^T M^T R = R^T M R = J. \tag{17}$$

Because inertia matrix M is symmetric itself ($M = M^T$).

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Differentiation of ξ_2

$$\xi_2 = k(q) \tag{18}$$

$$\dot{\xi}_2 = \frac{\partial k}{\partial q}\frac{dq}{dt} = \frac{\partial k}{\partial q}\dot{q} = \frac{\partial k}{\partial q}G\eta = S_2(q)\dot{\xi}_1 \tag{19}$$

where

$$S_2(q) = \frac{\partial k}{\partial q}GR \tag{20}$$

Therefore the following equation

$$\dot{\xi}_2 = S_2(q)\dot{\xi}_1. \tag{21}$$

The above equation is similar to the "kinematics".

Properties of linearised system

Model expressed in linearised coordinates has following properties:

1. Only the position is linearised.
2. The orientation can not be linearised.

Eccentric movement

The eccentric movement can be commonly observed because tracking is done for a point which is not the mass centre.

Unicycle (1/2)

Let's consider an unicycle platform – (2,0) platform. The kinematics is described as

$$\begin{aligned} \dot{x} &= v\cos\theta, \\ \dot{y} &= v\sin\theta, \\ \dot{\theta} &= \omega. \end{aligned}$$

or equivalently

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{pmatrix} = \dot{q} = \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{bmatrix} \begin{pmatrix} v \\ \omega \end{pmatrix} = G \begin{pmatrix} v \\ \omega \end{pmatrix} \tag{22}$$

Let's choose linearising functions $h(g)$ as

Unicycle (2/2)

$$h = \begin{pmatrix} h_1(q) \\ h_2(q) \end{pmatrix} = \begin{pmatrix} x + e\cos(\theta+\delta) \\ y + e\sin(\theta+\delta) \end{pmatrix} \tag{23}$$

$$R^{-1} = \frac{\partial h}{\partial q} G = \begin{bmatrix} 1 & 0 & -e\sin(\theta+\delta) \\ 0 & 1 & e\cos(\theta+\delta) \end{bmatrix} \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{bmatrix}$$

$$R^{-1} = \begin{bmatrix} \cos\theta & -e\sin(\theta+\delta) \\ \sin\theta & e\cos(\theta+\delta) \end{bmatrix} \tag{24}$$

Regularity condition

$$
\begin{aligned}
detR^{-1} &= e\cos\theta\cos(\theta+\delta) + e\sin\theta\sin(\theta+\delta) \\
&= e(\cos\theta\cos(\theta+\delta) + \sin\theta\sin(\theta+\delta)) \\
&= e\cos(-\delta) = e\cos\delta
\end{aligned}
$$

The condition is fulfilled when

$$detR^{-1} = e\cos\delta \neq 0,$$

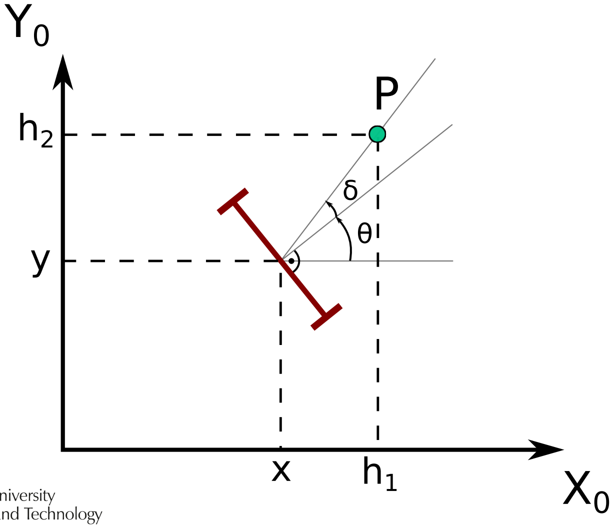
thus following two conditions should be met

$$e \neq 0 \wedge \delta \neq \frac{\pi}{2} + k\pi. \tag{25}$$

Guidance point P

Considering above the point P can not lay on the axle.

Geometric interpretation



Quiz (1/1)

Calculate group number as the rest from dividing the Student ID number by 4.

Example

Student ID number is 123456, thus the group is 0.
Take last 2 digits from Student ID number (56) and calculate the rest from dividing by 4 (56 % 4 = 0).

Write down your name, Student ID number and group.

Literature (1/1)

C. C. de Wit, B. Siciliano, and G. Bastin.
Theory of Robot Control.
Springer-Verlag London, 1996.